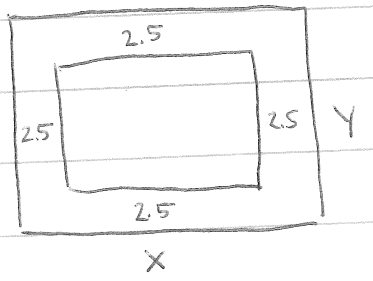
Optimization Quiz Review

In class practice:

1.



$xy = 942$

$x = \frac{942}{y}$

$x = \frac{942}{(30.692)}$

$x = 30.692$ ft.

$A_{max} = (x-5)(y-5)$

$A_{max} = xy - 5x - 5y + 25$

$A_{max} = \left(\frac{942}{y}\right)y - 5\left(\frac{942}{y}\right) - 5y + 25$

$A_{max} = 967 - 4710y^{-1} - 5y$

$A' = 4710y^{-2} - 5$

$0 = \frac{4710}{y^2} - 5$

$5 = \frac{4710}{y^2}$

$y = 30.692$ ft.

2. $C(q) = 120 + 2q^2$

$R(q) = 100q$

$P_{max} = 100q - (120 + 2q^2)$

$P_{max} = 100q - 120 - 2q^2$

$P' = 100 - 4q$

$0 = 100 - 4q$

$q = 25$ products

3. $y = \sqrt{x+3}$ to $(-2, 0)$

$y^2 = x + 3$

$y = \sqrt{(-2.5)+3}$

$y = .707$

$(-2.5, .707)$

$d_{min} = \sqrt{(x+2)^2 + (y-0)^2}$

$d_{min} = \sqrt{x^2+4x+4+y^2}$

$d_{min} = \sqrt{x^2+4x+4+(x+3)}$

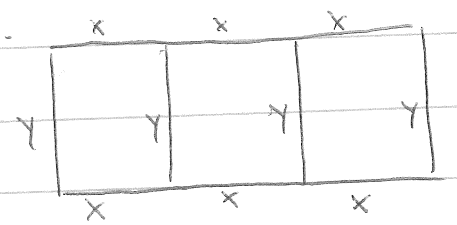
$d_{min} = (x^2+5x+7)^{1/2}$

$d' = \frac{1}{2}(x^2+5x+7)^{-1/2}(2x+5)$

$0 = \frac{2x+5}{2(x^2+5x+7)^{1/2}}$

$x = -2.5$

4.



$24(6x) + 24(2y) + 17(2y) = 2500$

$144x + 82y = 2500$

$144x = 2500 - 82y$

$x = \frac{2500 - 82y}{144}$

$x = \frac{2500 - 82(15.244)}{144}$

$x = 8.681$ ft.

$A_{max} = 3xy$

$A_{max} = 3y\left(\frac{2500 - 82y}{144}\right)$

$A_{max} = \frac{7500y - 246y^2}{144}$

$A_{max} = \frac{7500}{144}y - \frac{246}{144}y^2$

$A' = \frac{7500}{144} - \frac{492}{144}y$

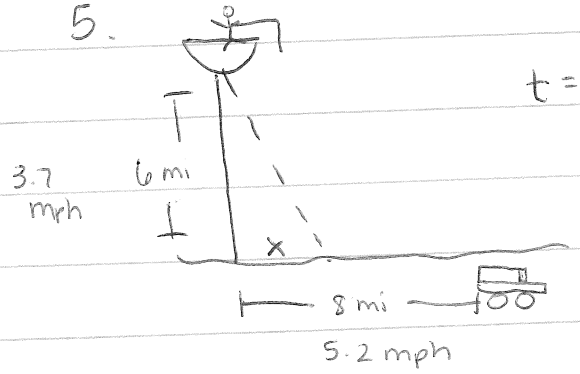
$0 = \frac{7500}{144} - \frac{492}{144}y$

$\frac{492}{144}y = \frac{7500}{144}$

$y = 15.244$ ft.

area of enclosure: 15.244 ft x 26.042 ft.

5.



$t = \frac{d}{r}$

$t_{min} = t_{boat} + t_{walk}$

$t_{min} = \frac{\sqrt{x^2+36}}{3.7} + \frac{8-x}{5.2}$

$t_{min} = \frac{1}{3.7}(x^2+36)^{1/2} + \frac{8}{5.2} - \frac{1}{5.2}x$

$t' = \frac{1}{7.4}(x^2+36)^{-1/2}(2x) - \frac{1}{5.2}$

$0 = \frac{x}{3.7(x^2+36)^{1/2}} - \frac{1}{5.2}$

$x = 6.076$ miles

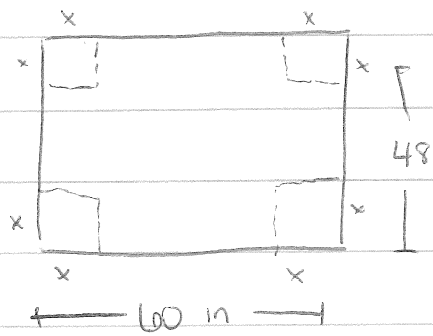
$t(0) = 3.160$ hrs.

$t(6.076) = 2.678$ hrs.

$t(8) = 2.703$ hrs.

He should row the boat to a point 6.076 mi from the point on the shore perpendicular to him.

6.



$l = 60 - 2x$

$w = 48 - 2x$

$h = x$

$V_{max} = lwh$

$V_{max} = (60-2x)(48-2x)x$

$V_{max} = (2880 - 120x - 96x + 4x^2)x$

$V_{max} = 2880x - 216x^2 + 4x^3$

$V' = 2880 - 432x + 12x^2$

$0 = 12(x^2 - 36x + 240)$

$x = 8.835, \cancel{27.165}$

$l = 60 - 2(8.835)$

$l = 42.33$ in

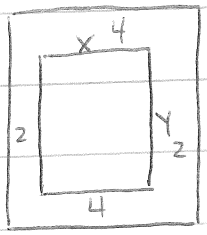
$w = 48 - 2(8.835)$

$w = 30.33$ in

$h = 8.835$ in

Take Home Practice:

1.



$xy = 50$

$x = \frac{50}{y}$

$x = \frac{50}{(10)}$

$x = 5$ in

$A_{max} = (x+4)(y+8)$

$A_{max} = xy + 8x + 4y + 32$

$A_{max} = (\frac{50}{y})y + 8(\frac{50}{y}) + 4y + 32$

$A_{max} = 50 + 400y^{-1} + 4y + 32$

$A_{max} = 400y^{-1} + 4y + 82$

$A' = -400y^{-2} + 4$

$0 = \frac{-400}{y^2} + 4$

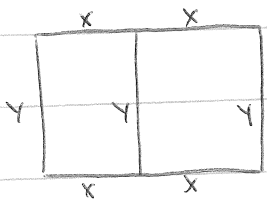
$\frac{400}{y^2} = 4$

$y = 10$ in

handbill:

9 in. x 18 in.

2.



$xy = 900$

$x = \frac{900}{y}$

a) $F_{min} = 4x + 3y$

$F_{min} = 4(\frac{900}{y}) + 3y$

$F_{min} = 3600y^{-1} + 3y$

$F' = -3600y^{-2} + 3$

$0 = \frac{-3600}{y^2} + 3$

$\frac{3600}{y^2} = 3$

$y = 34.641$ ft.

$x = \frac{900}{(34.641)}$

$x = 25.981$ ft.

25.981 ft x 34.641 ft.

b) $C_{min} = 2(4x) + 2(2y) + 1(y)$

$C_{min} = 8x + 5y$

$C_{min} = 8(\frac{900}{y}) + 5y$

$C_{min} = 7200y^{-1} + 5y$

$C' = -7200y^{-2} + 5$

$0 = \frac{-7200}{y^2} + 5$

$\frac{7200}{y^2} = 5$

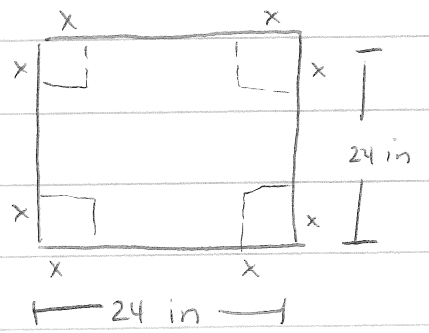
$y = 37.947$ ft.

$x = \frac{900}{(23.717)}$

$x = 23.717$

23.717 ft x 37.947 ft.

3.



$l = 24 - 2x$

$w = 24 - 2x$

$h = x$

$V_{max} = lwh$

$V_{max} = (24-2x)(24-2x)x$

$V_{max} = (576 - 96x + 4x^2)x$

$V_{max} = 576x - 96x^2 + 4x^3$

$V' = 576 - 192x + 12x^2$

$0 = 12(x^2 - 16x + 48)$

$x = 4$ ~~$x = 12$~~

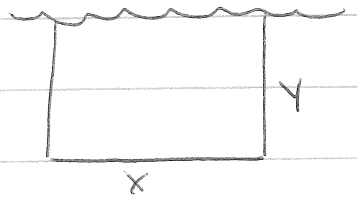
$l = 24 - 2(4)$

$l = 16$ in

$w = 16$ in

$h = 4$ in

4.



$xy = 180{,}000$

$x = \frac{180{,}000}{y}$

$F_{min} = x + 2y$

$F_{min} = 180000y^{-1} + 2y$

$F' = -180000y^{-2} + 2$

$0 = \frac{-180000}{y^2} + 2$

$\frac{180000}{y^2} = 2$

$y = 300$ ft.

$x = \frac{180000}{(300)}$

$x = 600$ ft.

300 ft x 600 ft.

5. $y = x^2 - 7$ to $(0,0)$

$x^2 = y + 7$

$x^2 = (-.5) + 7$

$x = \pm 2.546$

$d_{min} = \sqrt{(x)^2 + (y)^2}$

$d_{min} = \sqrt{(y+7) + y^2}$

$d_{min} = (y^2 + y + 7)^{1/2}$

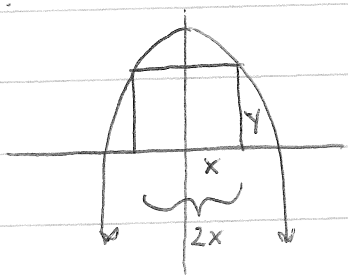
$d' = \frac{1}{2}(y^2 + y + 7)^{-1/2}(2y + 1)$

$0 = \frac{2y+1}{2(y^2+y+7)^{1/2}}$

$y = -.5$

$(2.546, -.5)$ & $(-2.546, -.5)$

6.



$y = -x^2 + 9$

$y = -(1.732)^2 + 9$

$y = 6$

$A_{max} = xy$

$A_{max} = x(-x^2 + 9)$

$A_{max} = -x^3 + 9x$

$A' = -3x^2 + 9$

$0 = -3(x^2 - 3)$

$x = \pm 1.732$

3.464×6